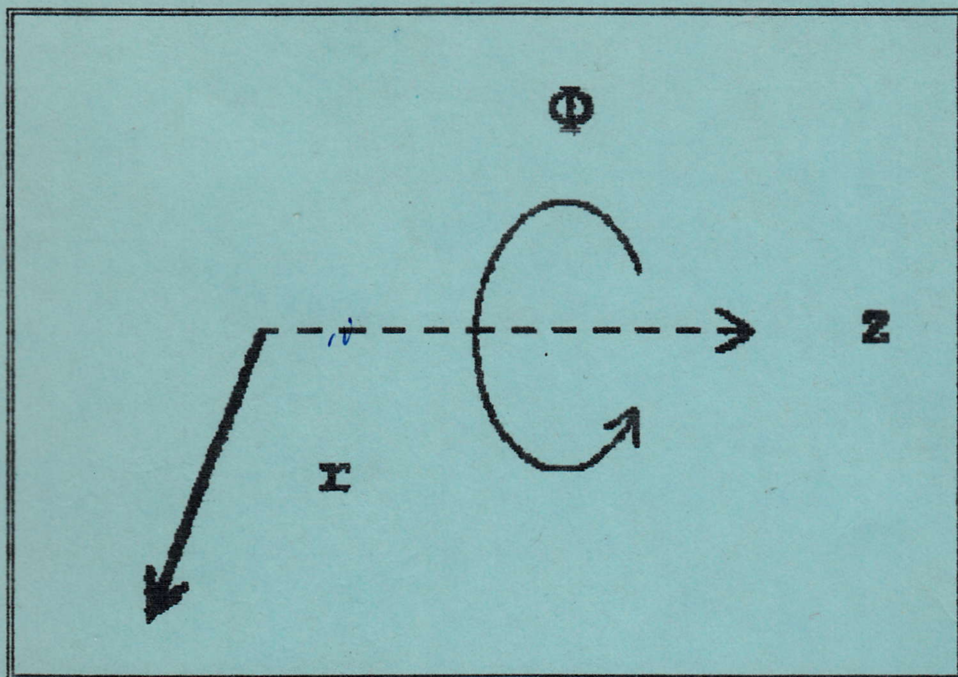

MOTIONAL ELECTRIC FIELDS ASSOCIATED WITH RELATIVE MOVING CHARGE

BY
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Kyle Aaron Klicker, 1986

ABSTRACT

The concept that the magnetic flux, induced by moving charge or an electrical current, moves with the charge carriers that induce it, is explored. This idea was promoted as late as the 1960's by W.J. Hooper and still remains a contested issue. Hooper claimed to have verified this experimentally and also identified some fundamental qualitative differences between types of electric fields distinguished by their origin. An analytical investigation of these claims has been undertaken.

This author has not been able to disprove Hooper's claims. It is established that there are three types of electric fields. The first due to a distribution of charge known as an electrostatic field. The other two are associated with the two types of electromagnetic induction. The first type of induction is known as flux cutting and is due to relative spatial motion with respect to magnetic flux. The electric field resulting from this type of induction is the motional electric field. This type of electric field has unique properties that separate it from the other two. Experimentally, it is confirmed that this electric field is immune to shielding due to the fact that magnetic (not electric) boundary conditions apply to it. Motional electric fields can also exist where the total magnetic field that induces it consists of non-zero components that sum to zero. The other type of induction is due to linking time changing magnetic flux.

Inclusion of the concept of magnetic flux moving with the current or charge carriers that induce it into classical electro-magnetic theory results in a small additional force between relative moving charge that is not predicted by classical EM theory. This difference is due to a motional electric field that surrounds all moving charge if the idea of moving magnetic flux is subscribed to. This term is dependent on the square of the relative velocity and is equivalent to the term generated by special relativity when applied to relative moving charge. Ampere electrodynamics also predicts the existence of this force. Consequently, three incompatible and fundamentally different models of EM effects yield the same results.

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CHAPTER 1

INTRODUCTION

Classical electromagnetic (EM) theory is a composite of pieces developed by such notables in the history of science as Faraday, Maxwell, Hertz, Lorentz and Einstein. The theory itself was originally developed from empirical results and experimental evidence. Even though the foundational development of the theory took place more than 150 years ago, the final chapters of EM theory have still not been written and this body of knowledge can still be an area for new discovery. As technology progresses and instrumentation becomes more sensitive, experimental evidence is obtained that still raises questions and paradoxes concerning the foundational aspects of EM theory. This is evidenced by the many parts of EM theory that alone are brilliant but when combined, do not fit into an easily understandable whole and do not always yield consistent results when applied to problems. Many authors [1,2,3,4,5] have pointed out inconsistencies and paradoxes in the theory and have demonstrated differences in results when applying one approach to a problem as compared to another.

Engineers and physicists have groped for a concise model and package that works for all of the vast EM phenomena. Maxwell's equations are the accepted answer to this need, but even they require the user to have an extensive prior knowledge of the results he is pursuing since they do not lend much physical insight into the mechanics of EM effects and can yield wrong results when applied indiscriminately. In addition, unless one is careful in applying these relationships [6,7], it is easy to exclude the common vXB term of the Lorentz force equation and this term may not be negligible.

One specific area of EM theory that still causes confusion [8,9] is the law of induction - the linking factor between electric and magnetic fields. It is commonly believed that Faraday's law of induction and the corresponding Maxwell equations describe all forms of induction and equates them, but this belief has been pointed out to be incorrect by many experimenters. In fact, there are two types of induction that must be treated separately [5,9]. The first is due to a time varying magnetic field and the second is due to relative spatial motion with respect to a magnetic field.

EM theory has its roots in experimental investigation and some of the cornerstones of EM theory such as Faraday's Law, The Biot-Savart version of Ampere's current Law and the Lorentz Force Equation are based purely on experimental results. Ampere was one of the first to develop a mathematical interpretation of these experiments. His experiments involved measuring the forces between currents and the results have been put into many forms. The most well known of these is the Biot-Savart Law that establishes the force between two currents as a function of their magnitudes, relative position and orientation. Faraday's law states that the induced electromotive force (e.m.f.) in a closed circuit is proportional to the time rate of change of magnetic flux it encloses. The Lorentz equation describes the force on a charge or an induced e.m.f. as a function of the magnitude of the electric field and magnetic field it sees and also the relative velocity with respect to the magnetic field.

One researcher, W.J. Hooper [10], probed deeply into the topic of classical EM theory and, after much experimentation, came to the conclusion that there are three different types of electric fields. One, due to a distribution of electric charge, and the other two are associated with the two types of induction.

His major interest was investigating the physical characteristics of the motional electric field that is associated with relative motion in space with respect to a magnetic field. Hooper distinguished between the electric field due to relative spatial motion with respect to a magnetic field and the electric field due to relative time motion with respect to a magnetic field. Motional electric fields are due to spatial not time motion with respect to a magnetic field. By adapting an empirical approach to his work, Hooper obtained evidence that the three types of electric fields have different physical characteristics and, therefore, should not be equated as is easy to do when using only mathematical models. Hooper demonstrated by experiment that motional electric fields are immune to shielding and can also exist where the total magnetic field is zero. In addition, he argued that magnetic flux is physically real and not just a mathematical model or a convenient way to describe the effects of moving charge.

More significant, Hooper interpreted the work of Cullwick [11] that assigned inertia and momentum to a current as supporting the idea that the magnetic flux (or field) induced by moving charge actually moves with the charge carriers. Hooper's premise that the magnetic flux associated with a current drifts along with the charge carriers composing the current is a simple one, but it has never been thoroughly investigated. Although never proven [3] before instrumentation was sensitive enough to measure it, it has some important ramifications and a new investigation is warranted. Charge itself can even be modeled by moving magnetic flux of spinning magnetic dipoles and a theory exists [12] that charge is moving flux. There seems to be no way to disprove that magnetic flux does not move with its source [12]. The most important ramification of the moving magnetic flux idea is that this assumption yields a motional electric field in the fixed reference frame of the current that induces the magnetic field. This will be true even if it is a dc current in a neutral conductor. Consequently, this idea proposes that a motional electric field is associated with all moving charge. To this author's knowledge, Hooper (with possibly one exception [13]) has been the only one to actually measure this effect. For moderate charge velocities, Special Relativity, when applied to the moving charge composing the current, supports his conclusion [14]. Besides Special Relativity, a field-free, non-relativistic version of the Ampere equation also supports Hooper's claims. It is ironic that these two approaches that avoid the use of magnetic fields give the same mathematical results as Hooper's theory that is centered on the physical reality of magnetic fields. The moving magnetic flux theory may have advantages, though, in its ability to characterize the force between relative moving charge as due to a motional electric field which by definition is a magnetic force (ie: $E=vXB$).

This author has set out to investigate the claim that magnetic flux moves with the charge that induces it. Originally an experimental approach was attempted. But limitations in available hardware and instrumentation needed to measure this small effect, has forced an analytical investigation of this idea. What will be shown in this paper is that Hooper's theory, with some clarifications the author proposes, is equivalent to a rigorous application of special relativity at moderate charge velocities (or to a second order approximation) and identically equivalent to an alternate field-free version of the Ampere equation [15]. Although the best way to 'get a hold of' an electromagnetic field is through its effects, the analytical investigation presented here does clarify certain issues and raise some significant points.

In Chapter 2, Hooper's experimental work is reviewed and evidence is presented that supports his claims. Chapter 3 analyzes various configurations of moving charge using three different formalisms of classical electrodynamics. The results generated in Chapter 3 are reviewed in Chapter 4 and a set of results is chosen as a baseline for further comparisons. Chapter 5 applies Hooper's, or the 'moving magnetic flux', approach to the same problems looked at in Chapter 3 and compares these results to the chosen baseline. A magnetic drift velocity that matches the baseline is derived. Advantages and disadvantages of the moving magnetic flux approach are then discussed in Chapter 6. In Chapter 7, conclusions about the work are drawn and experiments that would discriminate between a motional electric field effect and special relativity effects are suggested. It is shown that the motional electric field approach to determining the forces between relative moving charge is a valid one from a mathematical standpoint and may yield additional insight into the physical nature of the force between relative moving charge. Some key experiments that may differentiate between a motional electric field effect and a similar effect due to special relativity are suggested and described.

This work appears significant in light of recent work in the detection of magnetic monopoles [16], effects in field free regions as described by quantum gauge theory [17], and momentum possessed by static EM fields [18]. The moving magnetic flux idea may also help explain why dissimilar materials react differently to gravity that has now required the postulation of a small electric effect called 'Hyper-charge' or a fifth force that is a function of atom composition [19,20]. The linear motion of flux down a conductor may also help explain longitudinal propulsion associated with currents;an effect that has yet to be explained in terms of Lorentzian forces [21,13]. In fact, the incorporation of moving magnetic flux into EM field theory may help resolve the differences between Maxwellian field theory and Ampere-Neumann field free electrodynamics of materials.

CHAPTER 2

THE WORK AND THEORIES OF W. J. HOOPER

This investigation of motional electric fields associated with moving charge was stimulated by the work of W. J. Hooper in this same area. His contributions to the topic of EM theory involve the experimental investigation and description of motional electric fields, differentiating them from other types of electric fields, and his claim that a motional electric field is associated with all moving charge. Hooper's claims can be summed up by two premises. The first is that a motional electric field is physically different than an electrostatic field or an induced electric field due to a time changing magnetic field. His second premise is that a motional electric field is associated with all moving charge and this is due to the fact that the magnetic flux induced by moving charge moves with the charge.

Uniqueness of Motional Electric Fields

A regression at this point to define thoroughly motional electric fields is warranted. Simply stated, a motional electric field is the induced electric field due to relative motion with respect to magnetic flux [22]. It is described by the Lorentz force equation

$$E = vXB \qquad \text{E2.1.1}$$

where E is the induced motional electric field and v is the velocity of motion with respect to the magnetic field B.

Hooper stated and claimed to have proved experimentally that this motional electric field is different than the electric field that arises from a distribution of electric charge known as an electrostatic field and also different than the electric field due to time rate of change of a magnetic field.

In the classical sense, a motional electric field is the force per unit charge on a charge moving with respect to a magnetic field. It is considered a magnetic force and acts normal to both the magnetic field and the velocity of the charge. This is different from an electrostatic force that acts in line with the electric field. Another difference is that a magnetic force can not change the energy of the charge, only change its direction. This is even true in the case of a 'moving magnetic field' where there is a motional electric field produced. The magnetic field does not do work, but the source of the magnetic field or prime mover does work [23].

The differences between motional electric fields and those due to a time varying magnetic field are not always clear. Although a non-uniform moving magnetic field is mathematically equivalent to a time changing magnetic field,

$$\frac{dB}{dx} \cdot \frac{dx}{dt} = \frac{dB}{dt} \qquad \text{E2.1.2}$$

the physical characteristics of the effects they generate are different. The emf generated in a physically moving closed circuit can usually be described by Faraday's law since the amount of flux enclosed by the circuit is changing with time. The motional electric field is due to flux cutting while the electric field generated from a time changing magnetic field is due to flux linking. Although they are mathematically equivalent for certain geometries, Hooper claims that they are two different effects and should not be confused. This is in agreement with others who have rigorously investigated induction [8,9].

The work of Moon and Spencer on induction [9] helps to clarify this somewhat confusing issue. Their electrodynamic theory consists of modeling electromagnetic phenomena purely as forces between charges. This avoids the field concept all together. In their work, they show the equivalence of the flux cutting force described in the Lorentz force equation to a force due purely to relative motion between charge. The force due to flux linking is shown to be equivalent to a force between charge that is due purely to relative acceleration. From their work, it can be surmised that the flux cutting and flux linking are different phenomena since they stem from different fundamental physics. Flux cutting is equivalent to relative motion between charge while flux linking is due to relative acceleration between charge.

Another distinguishing characteristic of the motional electric field is its immunity to shielding. Hooper verified this with experiments. They consisted of showing that the motional electric field cannot be electrostaticly shielded by a faraday cage held at a fixed potential enclosing the detection device [10]. His experiments were also extended to magnetostatic shielding and his representative experiments concerning the shielding of motional electric fields have been duplicated and verified at Montana State University [24]. It was concluded that a motional electric field cannot be shielded by any common means. As long as magnetic flux is cut, an emf is produced independent of the material cutting the flux. This agrees with conclusions arrived at by Maxwell [25]. The only way to shield a material from a motional electric field is to use a magnetic shield (high μ material) around the source of the magnetic flux - in effect containing the magnetic flux at its source. When a magnetic shield is not around the source but around an object that is to be shielded, no shielding takes place since all the shield does is redirect the flux and the shielded area still cuts flux. These conclusions are not startling if one remembers that motional electric fields are a magnetic effect.

Another significant finding of Hooper was the measurement of real effects in field free regions. This may seem like a side-light to his work that is of interest here, but this effect is important and directly associated with motional electric fields. Hooper found that an analysis of the sum of the parts does not always equal the results of an analysis on the whole. A simple experiment that verifies this consists of subjecting a conductor to two different magnetic fields that are equal and opposite,

$$B_1 = -B_2 \qquad \text{E2.1.3}$$

and thus sum to zero,

$$B_1+B_2 = B_t = 0 \ . \qquad \text{E2.1.4}$$

If they also have equal and opposite relative velocity to a conductor,

$$v_1 = -v_2 \ , \qquad \text{E2.1.5}$$

the total E field is not equal to zero.

Not apparent at first glance, the correct result is obtained by application of the Lorentz force equation using simple superposition. Thus

$$E_t = v_1 X B_1 + v_2 X B_2 = 2vB\text{*}_n \ . \qquad \text{E2.1.6}$$

An incorrect value is obtained by using the total magnetic field.

$$E_t \neq (vXB_t = 0) \ . \qquad \text{E2.1.7}$$

In this case, even though the B fields cancel, the vXB effects add. A motional electric field due to flux cutting is generated even in an area where the total magnetic flux is zero. Hooper incorporated this effect in his experiment that generated a radial motional electric field using no moving parts.

Moving Magnetic Flux

Hooper's research and experimental work led him to draw the conclusion that the magnetic flux associated with a current actually moves with the charge carriers that compose the current. This assumption leads to the conclusion that a force exists between a dc current in a metallic conductor and an external stationary charge. This is equivalent to saying that a radial electric field surrounds a dc current even in a metallic conductor where charge neutrality (a balance between positive and negative charge numbers) is maintained within the conductor. This conclusion points out a interaction or equivalence of electrodynamic and electrostatic forces. This is in variance with the Biot-Savart Law and classical EM theory. In effect, this premise is equivalent to a type of 'dc induction'.

This force on stationary charge in the presence of a dc current (Figure 1) can be described by the Lorentz equation if one assumes that the magnetic flux due to a current,

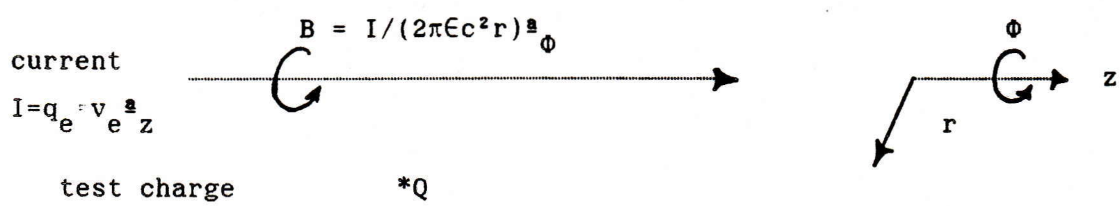
$$B = I/(2\pi \epsilon c^2 r)\hat{a}_\Phi \qquad \text{E2.2.1}$$

Where:
- I is the current magnitude in [amps]
- ϵ is the permittivity of free space [farads/meter]
- c is the velocity of light in free space [meters/sec.]
- r is the radial distance from the center of I [meters]

is drifting or moving with respect to the stationary charge at the drift velocity of the charge carriers that compose the current.

$$v_B = v_e = -v\hat{a}_z \qquad \text{E2.2.2}$$

Figure 1. Force on charge adjacent to dc current element.



The force on the charge then becomes

$$F = Q(v_B X B) = -v\hat{a}_z X I/(2\pi \epsilon c^2 r)\hat{a}_\Phi = Iv/(2\pi \epsilon c^2 r)\hat{a}_r \qquad \text{E2.2.3}$$

which is a finite value if Hooper's assumption that the magnetic flux has a finite velocity associated with it is accepted.

This idea that magnetic flux drifts along with its source is still an unresolved issue in EM theory. Although not exactly the same issue, it is worth mentioning that there has been an on-going debate initiated by Faraday with his axially symmetric 'disk' generator of whether the magnetic flux rotates with an axially symmetric source. This debate even intrigued Hertz [12]. Hooper's claim that magnetic flux moves with the charge carriers that comprise a current, has never been addressed with the same zeal as the rotating flux controversy. The most recent thought on the Faraday disk generator controversy is that whether the flux rotates with its source or not cannot be proven and either assumption yields the same results. This idea is suggested by Djuric [12] who has proposed a model of electric charge based on spinning magnetic dipoles. What most of these researchers have failed to realize and investigate thoroughly is one of the properties of the motional electric field, specifically its reaction to shielding. It seems that an experiment can be devised where shielding is used to distinguish between a motional electric field and the electric field of a charge redistribution caused by a motional electric field. Thus the controversy may be resolved once and for all. So far, it appears that most have overlooked the properties of motional electric fields. An example of this oversight is Djuric's model of charge based on spinning magnetic dipoles. It is sound mathematically, but fails to take into account the physical characteristics of the motional electric field. These characteristics would rend the charge of his model totally immune to shielding which is not in agreement with the known properties of electrostatic charge.

Experimental Work of W.J. Hooper

Hooper conducted many experiments that supported his claims concerning the uniqueness of the motional electric field, especially concerning the issue of shielding. His experiments consisted of a detection system that was usually a conductor and an ammeter that would measure the induced current in the conductor when it was passed through a magnetic field. This effect is well understood and is the principle of inducing an emf from cutting lines of magnetic flux. What is unique in Hooper's experiments is his investigation of various types of shielding. He was unable to shield the effects measured in his detection system by any type of electrostatic shielding such as a grounded faraday cage. Additionally, he could not shield his detector by employing any type of magnetic shielding such as high permeability iron. He concluded that the motional electric field due to relative motion between a conductor and a magnetic field was totally immune to shielding of any sort and penetrated all materials equally. These conclusions are in agreement with classical EM theory that defines a motional electric field rigorously as a magnetic force per charge, but his conclusions bring to attention certain characteristics of motional electric fields that are often overlooked.

The premise that there is a motional electric field associated with all moving charge lends itself to a straightforward experimental test. The concept is a simple one and the experiment is also simple in idea, but in practice has proven to be difficult strictly because the magnitude of the effect is so small.

The experiment Hooper used consisted of a source of moving charge and a detection system. The source consisted of a non-inductively (windings arranged so that the individual magnetic fields produced by each winding cancel and sum to zero) axial wound copper coil in a cylindrical configuration (4020 turns) energized by a variable power supply. The detection device consisted of an electrostaticly shielded (grounded faraday cage) cylindrical capacitor around the coil and an electrometer to measure the voltage induced in the capacitor by the motional electric field that surrounds the coil. His experiment and device is well described by his patents and papers [26,27]. A block diagram is shown in Figure 2.

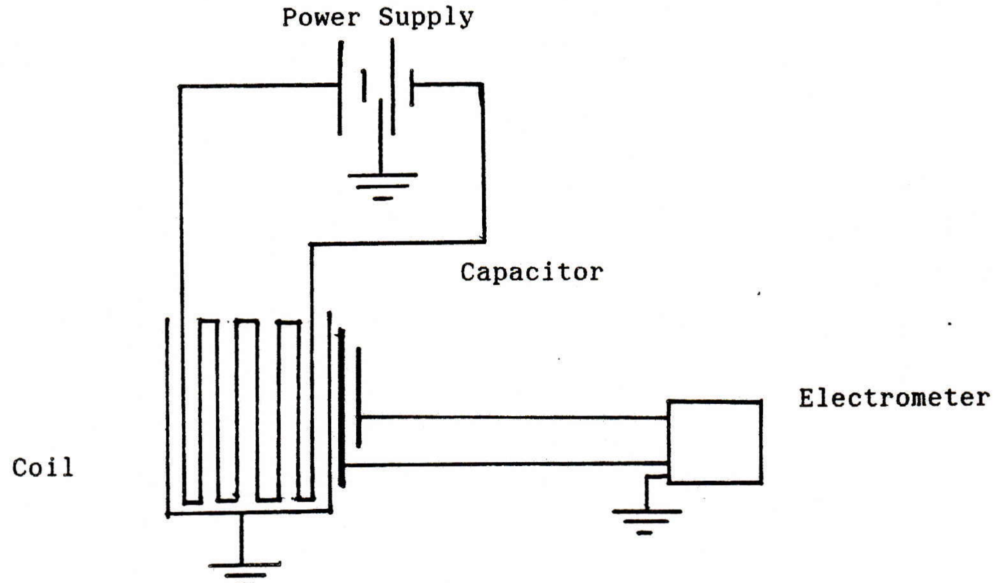


Figure 2. Block Diagram of Hooper's Experiment

Since Hooper's premise is contingent upon experimental proof, it is important to analyze a sample data point. The total magnetic field generated by Hooper's generator is

$$B = \frac{\mu N I}{2\pi r} \mathbf{a}_{\Phi} \qquad \text{E2.3.1}$$

Where:

$\mu = 4\pi \cdot 10^{-7}$ permeability of free space [Henrys/meter]
N = 4020 turns
I = current [Amps]
r = radial distance from center of coil [meters]
$\mathbf{a}_{\Phi}$ = unit vector in Φ direction [cylindrical co-ordinates]

The radial electric field surrounding the generator is calculated with the Lorentz equation (Equation 2.3.1). Rewriting this equation with the appropriate values of B and v gives

$$E = v\,\mathbf{a}_z \times \frac{\mu N I}{2\pi r}\,\mathbf{a}_\Phi = \frac{-v\mu N I}{2\pi r}\,\mathbf{a}_r \qquad \text{E2.3.2}$$

To determine the voltage difference that the cylindrical capacitor sees, the electric field must be integrated between the two plates of the capacitor.

$$V = \int_{r1}^{r2} \frac{-v\mu N I}{2\pi r}\,dr = \frac{-v\mu N I}{2\pi}\cdot\ln(r2/r1) \qquad \text{E2.3.3}$$

The distances from the center of the coil to the plates of the capacitors are, from Hooper's laboratory notes [28].

OD of inner cylinder = 0.10265 [Meters]

ID of outer cylinder = 0.10615 [Meters]

The electron drift velocity must also be obtained. This velocity depends on the conducting material, charge carrier mobility, charge carrier density, and the electric field applied to the material. For copper, at room temperature, a velocity of 0.02 meters per second is an accepted value. Hooper derived a value of 0.0176 meters/second [29,30] using Fermi-Dirac statistics and used this as a comparison when measuring drift velocity of the electrons in the copper coil with his generator. Using all of the proper values, equation 2.3.3 gives for a current of 30 Amps

$V = 14\ \mu$Volts

This compares favorably with some of his measured results and helps to confirm that the magnetic flux moves with the charge carriers. The result would be zero if the flux did not move.

Aspects of his experimental results that are significant are the characteristics of his data for a given test. He found that the voltage measured had a parabolic dependence on current for tests run at room temperature. If the electron drift velocity increases with current this relationship would result. This can be explained by assuming a linear relationship between applied voltage to the generator and current flow in the generator (this would be true for a constant temperature where the resistance of the coil did not change with applied voltage) and assuming that the number of charge carriers and their mobility is a constant for a given test and temperature. Since the drift velocity is equal to the electron mobility times the applied electric field, the drift velocity appears to increase linearly with current. The result of this is that the Iv term in equation 2.3.3 actually is qv^2 (i.e. parabolic in v) where v is linearly proportional to the current, I. Hence the ensuing parabolic relationship between measured voltage and applied current is obtained.

One other detail concerning Hooper's results needs to be checked and that is whether the voltage on the capacitor can actually be measured. The nature of this effect requires an electrostatic measurement of the voltage on the capacitor. This requires a high impedance electrometer. Hooper used a device that could measure charge as small as

$$Q = 10^{-16} \text{ coulombs}$$

The amount of charge accumulated on the capacitor must be calculated to determine if it exceeds this value. The capacitance of the detector is given by Hooper as 285 pico-farads. The charge accumulated on the capacitor for a voltage across its plates of 1 μVolt is 2.85 times above the limitations of the electrometer used. So it appears that the measurements made are physically possible.

The results of Hooper's experiments support his theory in that applying his assumption with the Lorentz equation and using a typical electron drift velocity for copper at room temperature a result is obtained that agrees with experiment. His results have a squared dependence on current that makes sense if the number of charge carriers remains a constant for varying currents and for the range of temperatures that this relationship held.

CHAPTER 3

CALCULATIONS OF FORCE BETWEEN MOVING CHARGE

To establish a baseline of results to compare to the moving magnetic flux model, the most accepted and commonly used analytical tools used to determine the force between relative moving charge and or current elements are explored here. Three different methods are employed to calculate the force between two current elements or two systems of moving charge (designated 1 and 2). The goal is to establish confidence in a set of results that the moving magnetic flux approach can be tested against in a later chapter. The three methods explored here include the classical method of the Biot-Savart law analysis of Ampere's experimental results (and it's equivalent, the Lorentz force equation), Special Relativity applied to the moving charge carriers that comprise the current, and a field free interpretation of Ampere's experiments derived by Moon and Spencer. From this point on, the three methods will be referred to as the Biot-Savart law, Special Relativity, and Moon and Spencer.

To cover the entire spectrum of combinations of moving charge and yet retain visibility and a realistic number of combinations of moving charge and calculations, the moving charge will be modeled as parallel in-line current elements of both a metallic and ionic nature, electron beams and stationary point charges. All configurations are chosen as co-linear charge distributions or currents since this eliminates much complexity of geometry, and makes the effects due purely to the motion of charge obvious.

In total, fifteen cases are examined using the three different analysis methods. Table 1 lists the fifteen combinations of moving charge that are investigated and Table 2 details a pictorial representation of the fifteen cases. The cases are designated by Roman numerals I through XV, each one being a unique configuration of moving charge in location 1 with respect to location 2. A metallic conductor is designated by a 'm' in Table 1 while an ionic conductor is designated with a 'i'. An electron beam is designated with an e and a stationary point charge by a '*'. Both situations of current 1 and 2 flowing in the same direction and opposing direction are examined. In Table 1, the opposing current configurations are designated by an 'o'.

Table 1. Combinations of Moving Charge Investigated

Case #	Descriptors: m=metallic, i=ionic, e=e-beam, *=stationary charge, o=opposing direction	
	Charge configuration 1:	Charge configuration 2:
I	m	m
II	m	m,o
III	m	*
IV	i	i
V	i	i,o
VI	i	*
VII	m	i
VIII	m	i,o
IX	m	e
X	m	e,o
XI	i	e
XII	i	e,o
XIII	e	e
XIV	e	e,o
XV	e	*

In Table 2 the linear charge densities are actually represented by a series of pluses (++++++++++) for positive charge (qp) and a series of minuses (-----------) for negative charge (qe). The magnitudes of the charge velocities are also given in Table 2 and their directions are shown with a -> or a <-. The resulting currents and their directions are also shown.

Table 2. Pictorial Representation of Moving Charge Combinations

Case #	Charge Configuration					
I	1:	v=0		+++++++++++++++++++++++		I=qv
		v	<-	-----------------------		->
	2:	v=0		+++++++++++++++++++++++		I=qv
		v	<-	-----------------------		->
II	1:	v=0		+++++++++++++++++++++++		I=qv
		v	<-	-----------------------		->
	2:	v=0		+++++++++++++++++++++++		I=qv
		v		-----------------------	->	<-
III	1:	v=0		+++++++++++++++++++++++		I=qv
		v	<-	-----------------------		->
	2:	v=0		+		I=0
IV	1:	$v=\frac{1}{2}v$		+++++++++++++++++++++++	->	I=qv
		$v=\frac{1}{2}v$	<-	-----------------------		->
	2:	$v=\frac{1}{2}v$		+++++++++++++++++++++++	->	I=qv
		$v=\frac{1}{2}v$	<-	-----------------------		->
V	1:	$v=\frac{1}{2}v$		+++++++++++++++++++++++	->	I=qv
		$v=\frac{1}{2}v$	<-	-----------------------		->
	2:	$v=\frac{1}{2}v$	<-	+++++++++++++++++++++++		I=qv
		$v=\frac{1}{2}v$		-----------------------	->	<-
VI	1:	$v=\frac{1}{2}v$		+++++++++++++++++++++++	->	I=qv
		$v=\frac{1}{2}v$	<-	-----------------------		->
	2:	v=0		+		I=0

Table 2. continued

VII	1:	v=0		++++++++++++++++++++++		I=qv
		v=v	<-	----------------------		->
	2:	v=½v		++++++++++++++++++++++	->	I=qv
		v=½v	<-	----------------------		->
VIII	1:	v=0		++++++++++++++++++++++		I=qv
		v=v	<-	----------------------		->
	2:	v=½v	<-	++++++++++++++++++++++		I=qv
		v=½v		----------------------	->	<-
IX	1:	v=0		++++++++++++++++++++++		I=qv
		v=v	<-	----------------------		->
	2:	v=v	<-	----------------------		I=qv
						->
X	1:	v=0		++++++++++++++++++++++		I=qv
		v=v	<-	----------------------		->
	2:	v=v		----------------------	->	I=qv
						<-
XI	1:	v=½v		++++++++++++++++++++++	->	I=qv
		v=½v	<-	----------------------		->
	2:	v=v	<-	----------------------		I=qv
						->
XII	1:	v=½v		++++++++++++++++++++++	->	I=qv
		v=½v	<-	----------------------		->
	2:	v=v		----------------------	->	I=qv
						<-
XIII	1:	v=v	<-	----------------------		I=qv
						->
	2:	v=v	<-	----------------------		I=qv
						->
XIV	1:	v=v	<-	----------------------		I=qv
						->
	2:	v=v		----------------------	->	I=qv
						<-
XV	1:	v=v	<-	----------------------		I=qv
						->
	2:	v=0		*		I=0

To yield results that are directly comparable between the fifteen cases, the currents (I) and line charge densities (q) are constant for all current elements; regardless of whether they are metallic or ionic conductors or electron beams in nature. This is accomplished by setting the current in the metallic conductor equal to $q \cdot v$ (where q is the linear charge density of the mobile electrons in the conductor and v is the electron drift velocity) and then matching this value in the ionic and electron beam models. So, for the electron beam, the electron velocity must be v and the linear charge density q to yield I=qv. On the other hand, for the ionic conductor, there are both positive and negative charge carriers that contribute to the current. For simplicity and symmetry, in the ionic current case, the linear charge densities of the mobile positive charge and negative charge are assumed equal. The drift velocities are also equal and in opposite directions and are one half the value of the drift velocity in the metallic and electron beam case. Consequently, the current in the ionic conductor is $I=(q \cdot \frac{1}{2}v)+(q \cdot \frac{1}{2}v)=q \cdot v$, the same result as the metallic conductor and electron beam current. A cylindrical coordinate system is used with currents flowing in the z direction, forces between current elements are in the r (radial) direction, and magnetic flux due to a current flowing in the z direction is in the Φ direction. The symmetrical geometry chosen always yields forces and electric fields in the radial direction and magnetic fields in the Φ direction, so in most calculations magnitudes of these quantities are used. Where vectors are used, they are designated by bold lettering. The accepted sign convention on current direction is used. That is, the direction of current is in the opposite direction of the flow of negative charge and in the same direction as the flow of positive charge.

Biot-Savart Version of Ampere's current law

The most widely used and accepted method to calculate the force between current elements is the Biot-Savart version of Ampere's current law [24]. It is stated as

$$F_{2/1} = \frac{\mu I_1 I_2}{4\pi} \int_2 \int_1 \frac{(a_r \times dl_1)}{R^2} \times dl_2 \qquad \text{E3.1.1}$$

Where:

$F_{2/1}$ is the total force on current 2 due to 1

μ is the permeability of free space

I_1 is current at 1: (constant in space)

I_2 is current at 2: (constant in space)

a_r is the unit vector along the distance vector between 1: and 2:

R is the distance between 1: and 2:

dl_1 is the incremental distance along current 1

dl_2 is the incremental distance along current 2

Clearly, this method yields a zero value for the cases where the charge distribution at location 1: or 2: does not yield a current. These are cases III, VI and XV in Table 2 where the charge distribution at 2: is a stationary point charge. Also, the lack of charge neutrality of cases involving an electron beam as a current source requires an additional force due to the electric field at 1:. This effects cases XIII, XIV and XV in Table 2.

Applying E3.1.1 on parallel currents of infinite length yields a force per unit length as

$$f_{2/1}= \mu I_1 I_2/(2\pi r) \text{ [Newtons/meter]} \qquad \text{E3.1.2}$$

in the radial direction where r is the distance between the two currents. For currents flowing in the same direction this force is attraction and is opposition for currents flowing in the opposite direction. Expressing μ in terms of the dielectric constant and the speed of light as $1/(\epsilon c^2)$ and for the case of

$$I_1 = I_2 = q\cdot v \qquad \text{E3.1.3}$$

the force per unit length between the two current elements becomes

$$f_{2/1} = q^2(v/c)^2/(2\pi\epsilon r) \qquad \text{E3.1.4}$$

Since cases XIII and XIV have a non-zero force due to their electrostatic charge, they require the addition of the opposing coulombic force $q^2/(2\pi\epsilon r)$ yielding :

$$\text{Case XIII: } f_{2/1} = q^2(1-(v/c)^2)/(2\pi\epsilon r) \qquad \text{E3.1.5}$$
$$\text{Case XIV: } f_{2/1} = q^2(1+(v/c)^2)/(2\pi\epsilon r) \qquad \text{E3.1.6}$$

Case XV has only the electrostatic term

$$F_{2/1} = Qq/(2\pi\epsilon r) \text{ [Newtons] .} \qquad \text{E3.1.7}$$

Another classical approach that yields the same results is the application of the Lorentz force equation to the charge configuration at 2:. The force on the charge at 2: depends on the magnitude of the electric and magnetic field of 1: (E1 and B1) and its relative velocity of 2: with respect to 1:.

$$F_{2/1} = q_2(E_1+v_{2/1}XB_1) \qquad \text{E3.1.8}$$

We can write the magnetic field B_1 as $I_1/(2\pi\epsilon c^2 r)$.

Expressing the current in terms of charge and velocity

$$q_2 v_{2/1} = I_2 \qquad \text{E3.1.9}$$

equation 3.1.8 can be written as

$$F_{2/1} = q_2 E_1 + I_1 I_2/(2\pi \epsilon c^2 r) = q_2 E_1 + q^2 (v/c)^2/(2\pi \epsilon r) \qquad \text{E3.1.10}$$

which is the same as the result of the Biot-Savart law.

In all cases in Table 1, except the last three, the electric field term E1 is zero and the only force is due to the relative motion of q2 with respect to the magnetic field of 1. This is a straightforward approach and yields the exact same results as the Biot-Savart law when we assume the magnetic field does not drift but is stationary in the reference frame of the inducing current. This method will be used later in Chapter 4 when the moving magnetic flux approach is investigated. Table 3 summarizes the results of the Biot Savart law and also includes the equivalent Lorentz force equation results when applied in the classical sense.

Table 3. Force Between Relative Moving Charge, Results of Biot-Savart Law and the Lorentz Force Equation.

Case #	Radial force between line charge, f [N/m] or line charge and point charge, F [N]	
	Biot-Savart Equation	Lorentz Force Equation
I	$-q^2(v/c)^2/(2\pi\epsilon r)$	$-q^2(v/c)^2/(2\pi\epsilon r)$
II	$q^2(v/c)^2/(2\pi\epsilon r)$	$q^2(v/c)^2/(2\pi\epsilon r)$
III	0	0
IV	$-q^2(v/c)^2/(2\pi\epsilon r)$	$-q^2(v/c)^2/(2\pi\epsilon r)$
V	$q^2(v/c)^2/(2\pi\epsilon r)$	$q^2(v/c)^2/(2\pi\epsilon r)$
VI	0	0
VII	$-q^2(v/c)^2/(2\pi\epsilon r)$	$-q^2(v/c)^2/(2\pi\epsilon r)$
VIII	$q^2(v/c)^2/(2\pi\epsilon r)$	$q^2(v/c)^2/(2\pi\epsilon r)$
IX	$-q^2(v/c)^2/(2\pi\epsilon r)$	$-q^2(v/c)^2/(2\pi\epsilon r)$
X	$q^2(v/c)^2/(2\pi\epsilon r)$	$q^2(v/c)^2/(2\pi\epsilon r)$
XI	$-q^2(v/c)^2/(2\pi\epsilon r)$	$-q^2(v/c)^2/(2\pi\epsilon r)$
XII	$q^2(v/c)^2/(2\pi\epsilon r)$	$q^2(v/c)^2/(2\pi\epsilon r)$
XIII	$q^2[1-(v/c)^2]/(2\pi\epsilon r)$	$q^2[1-(v/c)^2]/(2\pi\epsilon r)$
XIV	$q^2[1+(v/c)^2]/(2\pi\epsilon r)$	$q^2[1+(v/c)^2]/(2\pi\epsilon r)$
XV	$-Qq/(2\pi\epsilon r)$	$-Qq/(2\pi\epsilon r)$

Special Relativity Applied to Moving Charge

A more rigorous and exact approach in calculating the force between relative moving charge is the use of Special Relativity as applied to the electric fields of the charge. The increase in magnitude of an electric field due to relativity can be interpreted as an increase in mass of the charge carriers and assuming the charge to mass ratio is invariant. A second and more excepted interpretation considers the reference frame as shrinking and the charge density increasing. Either way, the same results are obtained; that the electric field of a given charge distribution when viewed from a reference frame that is moving with respect to the charge distribution increases in magnitude.

The general expression [5] of an electric field including the Lorentz transformation term is

$$E' = (E + vXB)/\sqrt{[1-(v/c)^2]} \qquad \text{E3.2.1}$$

where v is the velocity of the primed (') reference frame with respect to the non-primed reference frame. For velocities small compared to the speed of light, the correction transformation can be approximated by applying the binomial expansion theorem

$$(1-x)^{-n} = 1 + nx + n(n+1)x^2/2! + \ldots\ldots$$

In this case, $x=(v/c)^2$ and $n=\frac{1}{2}$ which yields to first order

$$1/\sqrt{[1-(v/c)^2]} \approx 1+\tfrac{1}{2}(v/c)^2 \qquad \text{E3.2.2}$$

This approximation will be used to simplify the calculation of the total electric field of a moving line charge.

To illustrate the method of Special Relativity , case I will be analyzed in detail and a general expression derived that can be applied to all the other cases.

The current of 1 and of 2 can be broken up into two line charges, one negative and one positive. This results in a four part problem to calculate the force between two currents. The total force per length on current element 2 due to 1 is

$$F_{2/1} = q_{p2}(E'_{p1/p2} + E'_{e1/p1}) + q_{e2}(E'_{p1/e2} + E'_{e1/e2}) \qquad \text{E3.2.3}$$

Where: q_{p2} is the positive line charge of 2

q_{e2} is the negative line charge of 2

$E'_{p1/p2}$ is the transformed E field due to the positive line charge of 1, q_{p1}, with respect to the positive line charge of 2, q_{p2}

$E'_{e1/p2}$ is the transformed E field due to the negative line charge of 1, q_{e1}, with respect to the positive line charge of 2

$E'_{p1/e2}$ is the transformed E field due to the positive line charge of 1, with respect to the negative line charge of 2, q_{e2}

$E'_{e1/e2}$ is the transformed E field due to the negative line charge of 1 with respect to the negative line charge of 2.

The process begins by calculating each of the transformed electric fields. The electric field of p1 with respect to p2 is

$$E'_{p1/p2} = [1+\tfrac{1}{2}(v_{p2/p1}/c)^2]\cdot E_{p1}$$

Where: $v_{p2/p1}$ is the velocity of p2 with respect to p1

E_{p1} is the electric field of line charge p1 and is equal to $q_{p1}/(2\pi\epsilon r)$.

This yields, upon combining

$$E'_{p1/p2} = q_{p1}[1+\tfrac{1}{2}(v_{p2/p1}/c)^2]/(2\pi\epsilon r) \ . \qquad \text{E3.2.4}$$

The electric field of e1 with respect to p2 is

$$E'_{e1/p2} = [1+\tfrac{1}{2}(v_{p2/e1}/c)^2]\cdot E_{e1}$$

Where: $v_{p2/e1}$ is the velocity of p2 with respect to e1

E_{e1} is the electric field of line charge e1 and is equal to $q_{e1}/(2\pi\epsilon r)$.

The combination yields

$$E'_{e1/p2} = q_{e1}[1+\frac{1}{2}(v_{p2/e1}/c)^2]/(2\pi\epsilon r) \quad \text{E3.2.5}$$

The transformed electric field of p1 with respect to e2 is

$$E'_{p1/e1} = [1+\frac{1}{2}(v_{e2/p1}/c)^2]\cdot E_{p1}$$

Where: $v_{e2/p1}$ is the velocity of e2 with respect to p1 .

Combining yields

$$E'_{p1/e2} = q_{p1}[1+\frac{1}{2}(\ _{e2/p1}/c)^2]/(2\pi\epsilon r) \quad \text{E3.2.6}$$

The transformed electric field of e1 with respect to e2 is

$$E'_{e1/e2} = [1+\frac{1}{2}(v_{e2/e1})^2]\cdot E_{e1}$$

Where: $v_{e2/e1}$ is the velocity of e2 with respect to e1 and

combining yields

$$E'_{e1/e2} = q_{e1}[1+\frac{1}{2}(v_{e2/e1}/c)^2]/(2\pi\epsilon r) \quad \text{E3.2.7}$$

Summing E3.2.4, E3.2.5, E3.2.6, and E3.2.7 in E3.2.3 results in the total force on 2 due to 1 as

$$\begin{aligned} F_{2/1} &= q_{p2}\{q_{p1}[1+\frac{1}{2}(v_{p2/p1}/c)^2]+q_{e1}[1+\frac{1}{2}(v_{p2/e1}/c)^2]\}/(2\pi\epsilon r) \\ &+ q_{e2}\{q_{p1}[1+\frac{1}{2}(v_{e2/p1}/c)^2]+q_{e1}[1+\frac{1}{2}(v_{e2/e1}/c)^2]\}/(2\pi\epsilon r). \end{aligned} \quad \text{E3.2.8}$$

This expression can now be applied to all cases, I-XV, by substituting in the appropriate values of

q_{p1}, q_{e1}, q_{p2}, q_{e2}, $v_{p2/p1}$, $v_{p2/e1}$, $v_{e2/p1}$, and $v_{e2/e1}$.

These values for case I are

$q_{p1}=q$ $\quad v_{p2/p1}=0$
$q_{e1}=-q$ $\quad v_{p2/e1}=v$
$q_{p2}=q$ $\quad v_{e2/p1}=v$
$q_{e2}=-q$ $\quad v_{e2/e1}=0$

Equation 3.2.8 then becomes

$$F_{2/1} = -q^2(v/c)^2/(2\pi\epsilon r)$$

For case II, the procedure is similar to case I, with the only difference being in the values of relative velocities (since the currents are flowing in different directions). Using the following values of relative velocities

$$\begin{array}{ll} q_{p1}=q & v_{p2/p1}=0 \\ q_{e1}=-q & v_{p2/e1}=v \\ q_{p2}=q & v_{e2/p1}=v \\ q_{e2}=-q & v_{e2/e1}=2v \end{array}$$

calculating E's, and combining in E3.2.2 yields

$$\begin{aligned} F_{2/1} &= q\{q/(2\pi\epsilon r)-q[1+\tfrac{1}{2}(v/c)^2]/(2\pi\epsilon r) \\ &\quad - q\{q[1+\tfrac{1}{2}(v/c)^2]/(2\pi\epsilon r)-q[1+2(v/c)^2]/(2\pi\epsilon r)\} \\ &= q^2(v/c)^2/(2\pi\epsilon r) \end{aligned}$$

These two results are the same as those calculated from the Biot-Savart law and are not surprising. What is interesting is the ability to describe a common, low velocity phenomena such as the force between current elements using relativity; specifically the correction factor that is usually assumed small enough to be ignored for velocities much less than the speed of light.

The next case involves the force that a stationary charge experiences from a current element. In this case,

$$\begin{array}{ll} q_{p1}=q & v_{p2/p1}=0 \\ q_{e1}=-q & v_{p2/e1}=v \\ q_{p2}=Q & v_{e2/p1}=0 \\ q_{e2}=0 & v_{e2/e1}=0 \end{array}$$

equation 3.2.3 becomes

$$F_{2/1} = Q\{q/(2\pi\epsilon r)-q[1+\tfrac{1}{2}(v/c)^2]/(2\pi\epsilon r)\} = -Qq(v/c)^2/(4\pi\epsilon r) \ .$$

This is a non-zero result and is not expected even though it is on the same order of magnitude as the force between current elements. It has been pointed out [14] that the neutrality of a current carrying conductor depends on the reference frame and this will become clear when case VI is analyzed.

Case IV involves determining the force between current elements that are in the same direction and ionic in nature. The relative velocities in this case are

$q_{p1}=q \qquad v_{p2/p1}=0$
$q_{e1}=-q \qquad v_{p2/e1}=v$
$q_{p2}=q \qquad v_{e2/p1}=v$
$q_{e2}=-q \qquad v_{e2/e1}=0$

which are identical to case I. Ssince the linear charge densities are the same, the result of equation 3.2.8 is also the same.

$$F_{2/1} = -q^2(v/c)^2/(2\pi\epsilon r) \ .$$

Case V involves ionic currents flowing in opposite directions and the relative velocities in this case are

$q_{p1}=q \qquad v_{p2/p1}=v$
$q_{e1}=-q \qquad v_{p2/e1}=0$
$q_{p2}=q \qquad v_{e2/p1}=0$
$q_{e2}=-q \qquad v_{e2/e1}=v$

which are symmetric to case IV and yield the same result but of opposite sign.

$$F_{2/1} = q^2(v/c)^2/(2\pi\epsilon r) \ .$$

Case VI is an ionic current and a stationary point charge. In this case the relative velocities are

$q_{p1}=q$ $v_{p2/p1}=\frac{1}{2}v$
$q_{e1}=-q$ $v_{p2/e1}=\frac{1}{2}v$
$q_{p2}=Q$ $v_{e2/p1}=0$
$q_{e2}=0$ $v_{e2/e1}=0$

and when used in equation 3.2.8, yield a zero result. In effect what has been accomplished here is that the reference frame in which a current carrying conductor is neutral has been determined. This same result would be obtained by choosing a metallic conductor such as case III, but finding the force on a point charge that was actually moving along with the electrons at half the electron velocity.

Cases VII and VIII are current elements of mixed nature one metallic and the other ionic. In case VII, the relative velocities are

$q_{p1}=q$ $v_{p2/p1}=\frac{1}{2}v$
$q_{e1}=-q$ $v_{p2/e1}=1.5v$
$q_{p2}=q$ $v_{e2/p1}=\frac{1}{2}v$
$q_{e2}=-q$ $v_{e2/e1}=\frac{1}{2}v$

and for case VIII

$q_{p1}=q$ $v_{p2/p1}=\frac{1}{2}v$
$q_{e1}=-q$ $v_{p2/e1}=\frac{1}{2}v$
$q_{p2}=q$ $v_{e2/p1}=\frac{1}{2}v$
$q_{e2}=-q$ $v_{e2/e1}=1.5v$

these values yield the same results as I and II or IV and V. Again these are the classical results for the force between current elements. As shown here, the nature of the current element for a charge neutral conductor has no bearing on the outcome of the result.

In contrast to this result, are the following cases IX - XV that have at least one current element modeled as a charged particle (electron) beam. For case IX, there is no positive charge distribution in 2, so the applicable relative velocities are

$q_{p1}=q$ $v_{p2/p1}=0$
$q_{e1}=-q$ $v_{p2/e1}=0$
$q_{p2}=0$ $v_{e2/p1}=v$
$q_{e2}=-q$ $v_{e2/e1}=0$

and the result becomes

$$f_{2/1} = -q^2(v/c)^2/(4\pi\epsilon r)$$

This result is exactly one half the value of the force between to current elements if they where metallic instead of one metallic and one an electron beam. Even though the magnitude and direction of the currents are identical to case I and the net charge of the electron beam has no effect since the other current element is neutral, they yield different results when using relativity.

Case X is the same as case IX but of opposite sign only.

Case XI and XII involve the force between a current element that is ionic in nature and an electron beam. Again the magnitude of the currents are the same as case IX and one would expect the same results. The relative velocities in this case are:

$q_{p1}=q$ $v_{p2/p1}=0$
$q_{e1}=-q$ $v_{p2/e1}=v$
$q_{p2}=q$ $v_{e2/p1}=1.5v$
$q_{e2}=-q$ $v_{e2/e1}=\frac{1}{2}v$

The result becomes

$$f_{2/1} = -q^2(v/c)^2/(2\pi\epsilon r)$$

for case XI and the opposite for case XII. This is surprising, since the results are not the same as that for a metallic conductor and electron beam that was obtained by using Special Relativity . What can be concluded here is that the forces caused by currents flowing in various media appear to be different. This will become even more clear by checking cases XIII - XV.

Since the positive charge in both 1 and 2 for cases XIII-XV is zero, there is only one relative velocity that applies and that is the relative velocity between the two negative charge distributions.

For case XIII: $v_{e2/e1}=0$.

This then becomes a purely static case and the result is

$$f_{2/1} = q^2/(2\pi\epsilon r) \ .$$

This case has been explored by others [4] as a paradoxical situation for which classical theory can yield confusing results. In case XIV, the electron beams are flowing in the opposite directions and the relative velocity becomes

$$v_{e2/e1}=2v$$

and the result becomes

$$f_{2/1} = q^2[1+2(v/c)^2]/(2\pi\epsilon r)$$

Again this is quite different from the results of the Biot-Savart law. The results of case XV is also different from the result of the Biot-Savart law and is

$$F_{2/1} = Qq[1+\tfrac{1}{2}(v/c)^2]/(2\pi\epsilon r)$$

Moon and Spencer Version of Ampere's Equation

The third method looked at is a version of Ampere's Force equation as developed by Moon and Spencer. It is a part of their field free electrodynamics that avoids the mathematical construct of electric fields. In addition, no corrections for relativity are needed in their model. Moon and Spencer's formulation gives the force between two charges as a sum of terms that depend on position, relative velocity, relative acceleration and the magnitude and rate of change of charge as

$$\frac{F}{Q2} = \mathbf{a}_r \frac{Q1}{4\pi\epsilon r^2}\left(\frac{v}{c}\right)^2 (1-1.5\cdot\cos^2\theta)$$
$$- \mathbf{a}_a \frac{Q1}{4\pi\epsilon c^2 r}\frac{dv}{dt}$$
$$- \mathbf{a}_r \frac{1}{4\pi\epsilon}\frac{d}{dr}\left(\frac{Q1}{r}\right) \qquad \text{E3.3.1}$$

The first term is the Ampere force, where v is the relative velocity between Q1 and Q2, θ is the angle between the unit vector in the direction of the relative velocity and the unit vector in the radial direction between the two charges, and r is the radial distance between Q1 and Q2. This is Ampere's original equation [15] and is equivalent to the Biot-Savart law. It includes the cross-product term (vXB) in the Lorentz force equation. The second term is due to the relative acceleration between Q1 and Q2 and is in a direction opposite to the direction of acceleration. This term is equivalent to the force on a charged particle in the presence of a time varying magnetic field and is equivalent to Faraday's law of induction. The third term contains the Coulombic force for constant Q and the equivalent of displacement current for a varying Q. This investigation is interested only in constant charge densities and velocities so the Ampere force and the coulombic force are the only terms in Moon and Spencer's formalism that apply. Stating the Ampere force in differential form gives

$$dF_{2/Q2} = dQ1(v/c)^2[1-1.5\cdot\cos\theta]/(4\pi\epsilon R^2) \qquad \text{E3.3.2}$$

where: v is the relative velocity between Q1 and Q2
θ is the angle between the velocity vector v and the direction vector between Q1 and Q2

For the case of a current modeled as an infinitely long line charge q [coulombs/meter] moving at velocity v in the z direction [25], equation 3.3.2 becomes

$$F_{2/Q2} = \int_{-\infty}^{\infty} q(v/c)[1-1.5\cdot\cos\theta]/(4\pi\epsilon R^2)\; dz \qquad \text{E3.3.3}$$

Since there are mixed variables (θ,R,z) within the integral sign that are not independent, they must be rewritten.

$$R^2 = r^2+z^2, \text{ and } z = r\cdot\cot\theta$$

Solving for dz and R in terms of the r and θ

$$R^2 = r^2[1+\cot^2\theta] \text{ and } dz = -r\cdot\csc^2\theta\; d\theta$$

From inspection, every charge element in the positive z direction has a corresponding charge element in the negative z direction that is moving in opposite directions. This has the effect of yielding a net zero force on the charge at location 2 (Table 2) in the z direction. Consequently we are only interested in the force in the r direction that is non-zero and it is equal to sinθ·dF2/Q2. Equation 3.3.3 can now be written with the proper substitutions and appropriate change in the limits of integration.

at $z=-\infty$ $\theta=\pi$ and at $z=\infty$ $\theta=0$

$$F_{2/1} = Q2\cdot\int_{\pi}^{0} q(v/c)^2 \frac{-r\cdot\sin\theta\csc^2\theta[1-1.5\cos^2\theta]}{4\pi\epsilon r^2[1+\cot^2\theta]}\, d\theta\ .$$

Using the relationship $\csc^2\theta = 1+\cot^2\theta$, this expression can be simplified and easily integrated to yield

$$F_{2/1} = Q2\, q(v/c)^2/(4\pi\epsilon r)$$

This expression can then be applied to the cases I-XV that are of interest.

Case I becomes the sum of four terms very similar to the approach used in relativity.

$$f_{2/1} = q_{p2}[q_{p1}(v_{p1/p2}/c)^2/(4\pi\epsilon r)+q_{e1}(v_{e1/p1}/c)^2/(4\pi\epsilon r)] + q_{e2}[q_{p1}(v_{p1/e2}/c)^2/(4\pi\epsilon r)+q_{e1}(v_{e1/e2}/c)^2/(4\pi\epsilon r)] \quad \text{E3.3.4}$$

This equation only includes the forces due to relative motion and for cases where there is a net charge the coulombic forces between charge distribution 1 and 2 must also be included. If this is done for the general case, an expression is obtained that is equivalent to equation 3.2.2, that of the first order Special Relativity approach with the only exception that the relative velocities are opposite. But since the relative velocities are squared, this has no effect on the results they yield. It can be concluded here that this approach is equivalent to that of Special Relativity for the cases of interest in this investigation. The identical results obtained from Special Relativity and the Moon and Spencer version of Ampere force law are tabulated in Table 4.

Table 4. Force Between Relative Moving Charge, Results of Special Relativity and Moon and Spencer.

Case #	Radial force between line charge, f [N/m] or line charge and point charge, F [N]	
	Special Relativity	Moon and Spencer
I	$-q^2(v/c)^2/(2\pi\epsilon r)$	$-q^2(v/c)^2/(2\pi\epsilon r)$
II	$q^2(v/c)^2/(2\pi\epsilon r)$	$q^2(v/c)^2/(2\pi\epsilon r)$
III	$-Qq(v/c)^2/(2\pi\epsilon r)$	$-Qq(v/c)^2/(2\pi\epsilon r)$
IV	$-q^2(v/c)^2/(2\pi\epsilon r)$	$-q^2(v/c)^2/(2\pi\epsilon r)$
V	$q^2(v/c)^2/(2\pi\epsilon r)$	$q^2(v/c)^2/(2\pi\epsilon r)$
VI	0	0
VII	$-q^2(v/c)^2/(2\pi\epsilon r)$	$-q^2(v/c)^2/(2\pi\epsilon r)$
VIII	$q^2(v/c)^2/(2\pi\epsilon r)$	$q^2(v/c)^2/(2\pi\epsilon r)$
IX	$-q^2(v/c)^2/(4\pi\epsilon r)$	$-q^2(v/c)^2/(4\pi\epsilon r)$
X	$q^2(v/c)^2/(4\pi\epsilon r)$	$q^2(v/c)^2/(4\pi\epsilon r)$
XI	$-q^2(v/c)^2/(2\pi\epsilon r)$	$-q^2(v/c)^2/(2\pi\epsilon r)$
XII	$q^2(v/c)^2/(2\pi\epsilon r)$	$q^2(v/c)^2/(2\pi\epsilon r)$
XIII	$q^2/(2\pi\epsilon r)$	$q^2/(2\pi\epsilon r)$
XIV	$q^2[1+2(v/c)^2]/(2\pi\epsilon r)$	$q^2[1+2(v/c)^2]/(2\pi\epsilon r)$
XV	$-Qq[1+\frac{1}{2}(v/c)^2]/(2\pi\epsilon r)$	$-Qq[1+\frac{1}{2}(v/c)^2]/(2\pi\epsilon r)$

CHAPTER 4

INTERPRETATION OF RESULTS AND CHOICE OF BASELINE

In the previous chapter, three different analysis techniques were employed to determine the force between fifteen combinations of relative moving charge. They consisted of the Biot-Savart law, Special Relativity , and the Moon and Spencer version of the Ampere equation. It was found that the two analysis methods based on relative moving charge (Special Relativity and Moon and Spencer) where equivalent to each other but yielded different results from the Biot-Savart law (a formalism based on currents and magnetic forces). Table 5 summarizes the two different results and will be useful for comparison.

The cases where the results differ between the Biot-Savart law and Special Relativity, are marked (*) for reference since they are of special interest.

Table 5. Force Between Relative Moving Charge, Comparison of Results of Chapter 3.

Case #	Radial force between line charge, f [N/m] or line charge and point charge, F [N]	
	Biot-Savart law	Special Relativity
I	$-q^2(v/c)^2/(2\pi\epsilon r)$	$-q^2(v/c)^2/(2\pi\epsilon r)$
II	$q^2(v/c)^2/(2\pi\epsilon r)$	$q^2(v/c)^2/(2\pi\epsilon r)$
* III	0	$-Qq(v/c)^2/(2\pi\epsilon r)$
IV	$-q^2(v/c)^2/(2\pi\epsilon r)$	$-q^2(v/c)^2/(2\pi\epsilon r)$
V	$q^2(v/c)^2/(2\pi\epsilon r)$	$q^2(v/c)^2/(2\pi\epsilon r)$
VI	0	0
VII	$-q^2(v/c)^2/(2\pi\epsilon r)$	$-q^2(v/c)^2/(2\pi\epsilon r)$
VIII	$q^2(v/c)^2/(2\pi\epsilon r)$	$q^2(v/c)^2/(2\pi\epsilon r)$
* IX	$-q^2(v/c)^2/(2\pi\epsilon r)$	$-q^2(v/c)^2/(4\pi\epsilon r)$
* X	$q^2(v/c)^2/(2\pi\epsilon r)$	$q^2(v/c)^2/(4\pi\epsilon r)$
XI	$-q^2(v/c)^2/(2\pi\epsilon r)$	$-q^2(v/c)^2/(2\pi\epsilon r)$
XII	$q^2(v/c)^2/(2\pi\epsilon r)$	$q^2(v/c)^2/(2\pi\epsilon r)$
* XIII	$q^2[1-(v/c)^2]/(2\pi\epsilon r)$	$q^2/(2\pi\epsilon r)$
* XIV	$q^2[1+(v/c)^2]/(2\pi\epsilon r)$	$q^2[1+2(v/c)^2]/(2\pi\epsilon r)$
* XV	$-Qq/(2\pi\epsilon r)$	$-Qq[1+\frac{1}{2}(v/c)^2]/(2\pi\epsilon r)$

* - results differ

As seen in Table 5, the results differ between the two sets for cases III, IX, X, XIII, XIV, and XV. At this point, it must be resolved as to which results to accept as the most physically correct. For 'true' current elements, cases I, II, IV, V, VII, VIII, (not electron beams or stationary charge) the results are all in agreement. From these cases, it can be concluded that the Biot-Savart law yields the same results as relativity for currents flowing in charge neutral mediums. For other cases, there are discrepancies and the appropriate value must be chosen. By looking at the physical nature of the different methods, some reasons can be established for the proper choice.

The physical interpretation of relativity or Moon and Spencer is that the force between current elements is due to the relative velocity between charge and can be interpreted as an electric force. In the Biot-Savart law, the force between current elements is modeled as a magnetic force. These two physical interpretations are different and can help explain the difference in the results. In the case of the Biot-Savart law for the configurations investigated here, when there isn't a magnetic field at both 1 and 2, such as cases III, IV, and V, the result is zero. In the other cases that differ, there exists a current and a magnetic field at both 1 and 2, and it appears that the nature of the current has an effect on the results which the Biot-Savart law does not address. The Biot-Savart law does not distinguish between different types of currents. It cannot differentiate between a current flowing in a metal, a plasma or a charge particle beam. To obtain the most accurate value of the force between current elements or moving charge it appears that the details must be known concerning the charge densities and their respective velocities. Relativity and Moon and Spencer require these details and thus generate more descriptive results. This is another example where an analysis of the parts (in this case the individual charges and there velocities) does not equal an analysis of the whole (the currents).

It appears that the first order application of relativity or the Moon and Spencer version of the Ampere force equation yield better results as they appear to be more rigorous in this regard. Consequently, the baseline or the results that will be used as a comparison to the moving flux model will be those generated by relativity or Moon and Spencer.

As far as the ease of use, the method of Special Relativity and Moon and Spencer require the appropriate relative velocities and charge densities and these may be difficult to obtain. Once these are obtained, a straight forward application of the formalism yields a solution. The Biot-Savart law requires only the currents to be known which can usually be directly determined. This requires less detail of information, but yields results that are not consistent with relativity. Relativity or the Moon and Spencer approach has the added advantage that one can use calculated results to predict charge velocities by working backward from known experimental data.

CHAPTER 5

MOVING MAGNETIC FLUX APPROACH

A baseline of the actual magnitude of forces between moving charge has been established and will now be compared to the results generated by the moving magnetic flux approach. The moving magnetic flux approach begins with the assumption that the magnetic flux surrounding a current actually moves with the charge carriers that comprise the current. Unlike the formalisms that the baseline is generated from, the moving magnetic flux model is based on magnetic forces and flux not just electric fields and their forces. A recent proponent of this theory, W. J. Hooper, did not try to determine the actual magnitude of the magnetic flux drift velocity in relation to the charge carrier velocity, but made the further assumption that these two velocities are equal. Starting with this assumption, a general expression of the moving magnetic flux approach will be derived that can be used to determine the force between relative moving charge for all of the fifteen cases explored in Chapter 3. This expression and the results it generates will be compared to the results generated by the first order application of Special Relativity that has been chosen as a baseline. Differences will be noted and if possible a value of magnetic drift velocity will be derived that better matches the baseline results.

The moving magnetic flux approach uses the Lorentz force equation, but not in the classical sense since a velocity is ascribed to the magnetic flux surrounding a current. Starting with the Lorentz force equation

$$f_{2/1} = q_2(E_1 + v_{2/B1} X B_1) \qquad \text{E5.1}$$

where:

E_1 is the electric field of 1

B_1 is the magnetic field of 1

$v_{2/B}$ is the velocity of q_2 with respect to B_1

and using the appropriate values of magnetic and electric fields and relative velocities, an equation can be derived that can be applied to all the cases being investigated. To obtain a general expression of equation 5.1 that can be used for all fifteen cases, the electric and magnetic fields of charge distribution 1 (Table 2) are broken up into their constituent parts and the charge in 2 is also described as a superposition of two different line charges. Rewriting, equation 5.1 becomes

$$f_{2/1} = q_{p2}[q_{p1} + v_{p2/Bp1} \cdot I_{p1/p2} + q_{e1} + v_{p2/Be1} \cdot I_{e1/p2}]/(2\pi c^2 \epsilon r)$$
$$+\ q_{e2}[q_{p1} + v_{e2/Bp1} \cdot I_{p1/e2} + q_{e1} + v_{e2/Be1} \cdot I_{e1/e2}]/(2\pi c^2 \epsilon r)$$

The currents can be further broken down and written in terms of the appropriate charge and velocity.

$$I_{p1/p2} = q_{p1} \cdot v_{p1/p2}$$
$$I_{e1/p2} = q_{e1} \cdot v_{e1/p2}$$
$$I_{p1/e2} = q_{p1} \cdot v_{p1/e2}$$
$$I_{e1/e2} = q_{e1} \cdot v_{e1/e2}$$

Since it is assumed that the magnetic flux is moving along with the charge in 1, the relative velocities of the charge in 2 with respect to the magnetic flux in 1 can be expressed as relative velocities between the charge in 2 and the charge in 1

$$v_{p2/Bp1} = v_{p2/p1}$$
$$v_{p2/Be1} = v_{p2/e1}$$
$$v_{e2/Bp1} = v_{e2/p1}$$
$$v_{e2/Be1} = v_{e2/e1}$$

Being aware of the definition of current direction and the sign generated by the cross-product term and also letting the charge carry its own sign, the magnitudes of the relative velocities can be used and the total expression becomes

$$f_{2/1} = q_{p2}\{q_{p1}[1+(v_{p2/p1}/c)^2]+q_{e1}[1+(v_{p2/e1}/c)^2]\}/(2\pi\epsilon r) + q_{e2}\{q_{p1}[1+(v_{e2/p1}/c)^2]+q_{e1}[1+(v_{e2/e1}/c)^2]\}/(2\pi\epsilon r) \quad \text{E5.2}$$

This is the same as equation 3.2.8, the general expression of Special Relativity, with the exception that the $(v/c)^2$ terms do not have a factor of one half multiplying them. Using equation 5.2 to calculate the force between 1 and 2 for the fifteen cases investigated yields the results tabulated in Table 6.

Table 6. Force Between Relative Moving Charge, Baseline Results Compared to Hooper version of Moving Magnetic Flux Approach.

Case #	Radial force between line charge, f [N/m] or line charge and point charge, F [N]	
	Special Relativity	Moving Mag. Flux Theory
I	$-q^2(v/c)^2/(2\pi\epsilon r)$	$-q^2(v/c)^2/(2\pi\epsilon r)$
II	$q^2(v/c)^2/(2\pi\epsilon r)$	$q^2(v/c)^2/(2\pi\epsilon r)$
* III	$-Qq(v/c)^2/(4\pi\epsilon r)$	$-Qq(v/c)^2/(2\pi\epsilon r)$
IV	$-q^2(v/c)^2/(2\pi\epsilon r)$	$-q^2(v/c)^2/(2\pi\epsilon r)$
V	$q^2(v/c)^2/(2\pi\epsilon r)$	$q^2(v/c)^2/(2\pi\epsilon r)$
VI	0	0
VII	$-q^2(v/c)^2/(2\pi\epsilon r)$	$-q^2(v/c)^2/(2\pi\epsilon r)$
VIII	$q^2(v/c)^2/(2\pi\epsilon r)$	$q^2(v/c)^2/(2\pi\epsilon r)$
* IX	$-q^2(v/c)^2/(4\pi\epsilon r)$	0
* X	$q^2(v/c)^2/(4\pi\epsilon r)$	$q^2(v/c)^2/(\pi\epsilon r)$
XI	$-q^2(v/c)^2/(2\pi\epsilon r)$	$-q^2(v/c)^2/(2\pi\epsilon r)$
* XII	$q^2(v/c)^2/(2\pi\epsilon r)$	$q^2(v/c)^2/(4\pi\epsilon r)$
XIII	$q^2/(2\pi\epsilon r)$	$q^2/(2\pi\epsilon r)$
* XIV	$q^2[1+2(v/c)^2]/(2\pi\epsilon r)$	$q^2[1+4(v/c)^2]/(2\pi\epsilon r)$
* XV	$-Qq[1+\frac{1}{2}(v/c)^2]/(2\pi\epsilon r)$	$-Qq[1+(v/c)^2]/(2\pi\epsilon r)$

* - Results differ

Since equation 5.2 is different than equation 3.2.8, it stands to reason that there are some differences between the baseline values and those generated by the moving magnetic flux model. The cases that are the same, I, II, IV, V, VII, VIII are all configurations where the system at both 1 and 2 is a true current as commonly accepted. This indicates that for these cases, the magnetic flux velocity is arbitrary and has no effect on the outcome as long as both current 1 and 2 are flowing in materials where charge neutrality is maintained. The cases that do differ are off by a factor of two. From these results, it can be concluded that the magnetic flux velocity cannot equal the charge velocity if one is to obtain the same results as the baseline. It must be stressed here that it is assumed that the baseline is correct, and only experimental evidence can determine if this is true.

If the definition of current as I=qv is used, the only way these results (cases III, IX, XII, XIV, XV) can be matched is to assume a magnetic flux drift velocity of one half the charge carrier velocity. This may seem like an arbitrary action, but one must remember that formulas such as the Biot-Savart and the Moon and Spencer version of Amperes current law were derived to match empirical information. Consequently, this is a valid model as long as physical phenomena is described by it accurately.

Rewriting equation 5.2 with the added assumption that the magnetic drift velocity is identically one half the charge carrier drift velocity, the following is obtained

$$\begin{aligned} F_{2/1} = {} & q_{p2}\{q_{p1}[1+\tfrac{1}{2}(v_{p2/p1}/c)^2]+q_{e1}[1+\tfrac{1}{2}(v_{p2/e1}/c)^2]\}/(2\pi\epsilon r) \\ & + q_{e2}\{q_{p1}[1+\tfrac{1}{2}(v_{e2/p1}/c)^2]+q_{e1}[1+\tfrac{1}{2}(v_{e2/e1}/c)^2]\}/(2\pi\epsilon r). \end{aligned} \qquad \text{E5.3}$$

This relationship is equivalent to the Special Relativity approximation and will yield identical results to all the cases looked at. This moving magnetic flux model matches relativity and furthermore may actually be a physical explanation for what is happening. To retain the formulation in terms of magnetic fields and the cross product term, we can define the magnetic field velocity of 1 as the weighted average of all the charge carrier velocities in 1 with respect to 2 that contribute to the current, summed vectorially and divided by 2. Expressed mathematically as

$$v_{m/q} = [\Sigma n_i \cdot v_{i/q}]/2n_t \qquad \text{for } n=0 \text{ to } i$$

where:

$v_{m/q}$ = magnetic flux drift velocity

n_i = fractional composition of charge i

$v_{i/q}$ = velocity of charge i

n_t = total charge = Σn_i

In addition, one must be careful to use the values of currents in 1 that the charge in 2 actually sees, not the current measured in some rest frame. This only has an effect in cases where the total charge distribution in 1 is not neutral, such as an electron beam. For a neutral conductor the current is the same in all reference frames.

Table 7 is a summary of the results obtained for all fifteen cases using the different analysis methods that have been explored. It is interesting that there can be such variation among the different tools of EM theory.

Table 7. Force Between Relative Moving Charge, Summary of Results.

Moving charge config. Metallic conductor = m Ionic conductor = i Electron beam = e Current element -> Stationary charge *	Radial force between line charge, $f_{2/1}$ [N/m] or line charge and point charge, $F_{2/1}$ [N]			
	Biot-Savart Eq. or Lorentz Eq. with $v_m = 0$	Baseline, Moon and Spencer or Relativity	Moving Magnetic Flux, Lorentz Eq. with $v_m = v_q$	Moving Magnetic Flux, Lorentz Eq. with $v_m = \frac{1}{2}v_q$
I m -> m ->	$\frac{-q^2(v/c)^2}{2\pi\epsilon r}$	$\frac{-q^2(v/c)^2}{2\pi\epsilon r}$	$\frac{-q^2(c/v)^2}{2\pi\epsilon r}$	$\frac{-q^2(v/c)^2}{2\pi\epsilon r}$
II m -> m <-	$\frac{q^2(v/c)^2}{2\pi\epsilon r}$	$\frac{q^2(v/c)^2}{2\pi\epsilon r}$	$\frac{q^2(v/c)^2}{2\pi\epsilon r}$	$\frac{q^2(v/c)^2}{2\pi\epsilon r}$
III m -> *	0	$\frac{-Qq(v/c)^2}{4\pi\epsilon r}$	$\frac{-Qq(v/c)^2}{2\pi\epsilon r}$	$\frac{-Qq(v/c)^2}{4\pi\epsilon r}$
IV i -> i ->	$\frac{-q^2(v/c)^2}{2\pi\epsilon r}$	$\frac{-q^2(v/c)^2}{2\pi\epsilon r}$	$\frac{-q^2(v/c)^2}{2\pi\epsilon r}$	$\frac{-q^2(v/c)^2}{2\pi\epsilon r}$
V i -> i <-	$\frac{q^2(v/c)^2}{2\pi\epsilon r}$	$\frac{q^2(v/c)^2}{2\pi\epsilon r}$	$\frac{q^2(v/c)^2}{2\pi\epsilon r}$	$\frac{q^2(v/c)^2}{2\pi\epsilon r}$
VI i -> *	0	0	0	0
VII m -> i ->	$\frac{-q^2(v/c)^2}{2\pi\epsilon r}$	$\frac{-q^2(v/c)^2}{2\pi\epsilon r}$	$\frac{-q^2(v/c)^2}{2\pi\epsilon r}$	$\frac{-q^2(v/c)^2}{2\pi\epsilon r}$
VIII m -> i <-	$\frac{q^2(v/c)^2}{2\pi\epsilon r}$	$\frac{q^2(v/c)^2}{2\pi\epsilon r}$	$\frac{q^2(v/c)^2}{2\pi\epsilon r}$	$\frac{q^2(v/c)^2}{2\pi\epsilon r}$
IX m -> e ->	$\frac{-q^2(v/c)^2}{2\pi\epsilon r}$	$\frac{-q^2(v/c)^2}{4\pi\epsilon r}$	0	$\frac{-q^2(v/c)^2}{4\pi\epsilon r}$
X m -> e <-	$\frac{q^2(v/c)^2}{2\pi\epsilon r}$	$\frac{q^2(v/c)^2}{4\pi\epsilon r}$	$\frac{q^2(v/c)^2}{\pi\epsilon r}$	$\frac{q^2(v/c)^2}{4\pi\epsilon r}$
XI i -> e ->	$\frac{q^2(v/c)^2}{2\pi\epsilon r}$	$\frac{q^2(v/c)^2}{2\pi\epsilon r}$	$\frac{q^2(v/c)^2}{2\pi\epsilon r}$	$\frac{q^2(v/c)^2}{2\pi\epsilon r}$
XII i -> e <-	$\frac{q^2(v/c)^2}{2\pi\epsilon r}$	$\frac{q^2(v/c)^2}{2\pi\epsilon r}$	$\frac{q^2(v/c)^2}{4\pi\epsilon r}$	$\frac{q^2(v/c)^2}{2\pi\epsilon r}$
XIII e -> e ->	$\frac{q^2[1-(v/c)^2]}{2\pi\epsilon r}$	$\frac{q^2}{2\pi\epsilon r}$	$\frac{q^2}{2\pi\epsilon r}$	$\frac{q^2}{2\pi\epsilon r}$
XIV e -> e <-	$\frac{q^2[1+(v/c)^2]}{2\pi\epsilon r}$	$\frac{q^2[1+2(v/c)^2]}{2\pi\epsilon r}$	$\frac{q^2[1+4(v/c)^2]}{2\pi\epsilon r}$	$\frac{q^2[1+2(v/c)^2]}{2\pi\epsilon r}$
XV e -> *	$\frac{-Qq}{2\pi\epsilon r}$	$\frac{-Qq[1+\frac{1}{2}(v/c)^2]}{2\pi\epsilon r}$	$\frac{-Qq[1+(v/c)^2]}{2\pi\epsilon r}$	$\frac{Qq[1-\frac{1}{2}(v/c)^2]}{2\pi\epsilon r}$

CHAPTER 6

MOTIONAL ELECTRIC FIELDS ASSOCIATED WITH THE MOVING MAGNETIC FLUX MODEL

In Chapter 2, the unique properties of the motional electric field associated with 'cutting lines of magnetic flux' was described and later in Chapter 5 it was shown that a moving magnetic flux approach can be used to model the force between relative moving charge. This moving magnetic flux approach gives the same results when applied to configurations of relative moving charge that a exhaustive application of special relativity does. There may be some advantages to the moving magnetic flux model over other theories such as special relativity or Moon and Spencer, due to the peculiar qualities that are associated with the motional electric fields that arise in the moving magnetic flux model.

Motional electric fields, although having the same mathematical expression as static electric fields or those due to transformer action, are physically different and can easily be distinguished from the other types. The assumption that the magnetic flux associated with moving charge actually moves with the charge, results in a motional electric field surrounding the charge. This motional electric field is the same magnitude as the increase in the static electric field that relativity predicts due to the charge's motion. The moving magnetic flux model can be concluded to have advantages over the others formalisms that have been looked at because of the nature of the motional electric field that it predicts. The moving flux model is more intuitive than relativity in its description of physical effects.

The motional electric field is immune to electrostatic shielding, since by definition, it is a magnetic phenomena. This was documented by Hooper [11] and also verified at Montana State University [24]. Since motional electric fields arise from relative motion with respect to magnetic flux, any material that magnetic flux can penetrate will not shield a motional electric field. What is suggested by the results in Chapter 5 is that not only is a motional electric field associated with relative motion with respect to magnetic flux, but that there also is a motional electric field associated with all moving charge. This motional electric field is the same magnitude as a correction due to relativity. The only way that this motional electric field (due purely to relative moving charge) can occur is if the magnetic flux associated with the moving charge moves with the charge. By comparing the motional electric field to a set of results accepted as correct, the velocity of the moving flux was determined to be exactly one half the velocity of the moving charge. Although the moving magnetic flux model only matches the results of Special Relativity to a second order approximation, it does match a version of the Ampere equation by Moon and Spencer exactly. In addition, Moon and Spencer claim that charge and mass are invariant and Special Relativity is not needed [14]. It can then also be surmised that the moving magnetic flux model does not need to be corrected for relativity and assumes invariant charge and mass.

It is commonly thought that the force between currents is a magnetic one and thus cannot be shielded electrostaticly. Yet in

Chapter 3, it was shown that the force between currents (even two that are in metallic conductors) can be predicted using Special Relativity applied to the electric fields of the charge of the currents. The question arises whether a correction due to relativity is shieldable or if the correction due to relativity is actually the common phenomena of flux cutting. This question can only be answered with an exhaustive experimental investigation. What is suggested is a reworking of Ampere's experiments on forces between current elements with the added complication of shielding one or the other of the currents with various materials held at fixed and floating potentials. In addition, the current elements in different media, such as metals, plasmas, etc. should be investigated.

More fundamentally, the motional electric field surrounding a current should be measured directly. Measuring this field is exactly what Hooper appears to have done. The experimental setup required to accomplish this would be similar to the apparatus Hooper used to measure the motional electric field surrounding his generator. A source of moving charge to generate the motional electric field and a detection system to measure the motional electric field needed. Hooper's source of moving charge was the conduction electrons in a copper coil. Higher velocity charge such as in charged particle beams or conduction electrons in superconducting materials would have the advantage over Hooper's experiment of producing motional electric fields of greater intensity. This would reduce the sensitivity of the instrumentation needed to measure the motional electric field. Hooper's detection system consisted of a capacitor that was charged by the motional electric field and an electrometer to measure the voltage on the capacitor. This system can be used or a device (such as a torque pendulum [13]) to measure force on a charged particle could be used to detect the motional electric field.

A project was initiated at Montana State University that would measure the motional electric field surrounding an electron beam. An experiment with an electron beam as the source of moving charge and a shielded capacitor and electrometer as the detector was proposed. Limitations in size of the electron beam that could be generated and the sensitivity of equipment (specifically an electrometer) prevented the completion of this experiment.

Even more interesting than proving the existence of this electric field that is associated with moving charge, is the investigation of its properties. It is suggested by the moving magnetic flux hypothesis that it is a motional electric field, but relativity does not give any insight into the nature of the effect other than it does exist. Consequently, an experimental verification should be designed with the added intention of determining its physical characteristics. Experiments that study different types of shielding are critical.

CHAPTER 7

CONCLUSIONS AND SIGNIFICANCE OF WORK

Even though the theory of electromagnetism is considered mature and well established, there are still some unresolved issues in the theory. One such issue has been explored here, the premise of whether the magnetic field surrounding a moving charge actually moves with the charge. The most recent and avid proponent of this idea was W. J. Hooper. His claim, that the magnetic flux induced by moving charge moves with the charge, results in a motional electric field associated with all moving charge. The motional electric field is the direct result of the type of induction associated with cutting lines of magnetic flux or spatial movement with respect to magnetic flux. This induction is different than the type due to linking time changing (motion in time) magnetic flux. The two types of induction generate different types of electric fields.

The motional electric field, due only to the motion of charge (if the induced magnetic flux moves with the charge), has been shown here to be contrary to classical EM theory, but is the same magnitude as a correction factor due to relative motion that the theory of Special Relativity predicts. In addition, an alternative version of Ampere's current law also predicts an electric field associated with moving charge that is solely dependent on the charge's magnitude and velocity. The equivalence between a force predicted between relative moving charge by both relativity and a form of Ampere electrodynamics raises some serious questions about relativity. If relativity is used to calculate the force between current elements, the same results are obtained as measured by Ampere and predicted by the electrodynamic models generated from his work. Possibly, the force between current elements is a verification of relativity and Ampere electrodynamics are inconsistent with relativity. Or relativity is not the appropriate explanation for electrodynamic forces in materials. Either way, this issue needs to be explored.

A complete agreement between the 'moving magnetic flux premise', Special Relativity to second order, and the Moon and Spencer version of Ampere's law is obtained when the magnetic flux drift velocity is assumed to be one half the charge carrier drift velocity in the moving magnetic flux model. It is significant that a relativistic correction can be predicted by the moving magnetic flux idea as well as a version (Moon and Spencer) of Ampere's current law. The differences between the moving magnetic flux model and Special Relativity or Moon and Spencer are qualitative in nature and have to be determined experimentally. The electric field or force on charge associated with relative motion with respect to a magnetic field has characteristics that distinguish it from static electric fields associated with charge distribution or even the induced electric field due to time changing magnetic fields. Since motional electric fields have a unique 'finger print', it may be possible to devise an experiment to distinguish between a motional electric field or a electric field due to relativity transformation (that would be electrostatic in nature) on an electric field.

Some of the unique properties of motional electric fields are their reaction to different materials and types of shielding. They are immune to shielding configurations such as faraday cages held at fixed potentials. Motional electric fields can also be generated in regions where the total magnetic field can sum to zero. This effect can be used as another determining criteria in experiments in determining the nature of an EM effect.

It can be concluded that the moving magnetic flux idea that associates a motional electric field with all moving charge raises some interesting points concerning foundational EM theory. It's incorporation into classical EM theory may preclude the necessity of correcting for relativity. In fact, for the cases explored here if both moving magnetic flux and relativity are considered, the results obtained would be in disagreement with those considered correct. The concept of moving magnetic flux may also help to dispel some of the paradoxes of electrodynamic forces especially in areas that there is disagreement between classical Maxwellian field theory and Ampere electrodynamics applied to materials.

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6. Penfield, P. and Haus, H. Electrodynamics of Moving Media. MIT Press, Cambridge, Mass. (1967) chp. 3.
Comments: The accepted authority on electrodynamics.

7. Rosen, N. "Some Remarks on Faraday's Law," American Journal of Physics. 50(11), Nov., 1982. p. 974.
Comments: Rosen proposes incorporating the vXB term directly in

the definition of electric field used in Faraday's law.

8. Cohn, G. "Electromagnetic Induction," Electrical Engineering. (May, 1949) p.441-447.
Comments: Presents an approach to EM theory based on using the right type of induction description for a specific problem.

9. Moon, P. and Spencer, D. "On Electromagnetic Induction," Ibid., (Sept.,1955) p. 213-226.
Comments: Five problems are solved by four different expressions of the law of induction. A proposed new electrodynamics is shown to be the only theory to yield the correct results for all five examples.

10. Hooper, W. New Horizons in Electric, Magnetic and Gravitational Field Theory. Electrodynamic Gravity Inc., Akron, Ohio. (1974).

11. Cullwick, E. Electromagnetism and Relativity. Longsman Green and Co.
Comments: Cullwick does not actually say that the magnetic flux associated with a current moves with the charge that makes up the current, but he does subscribe linear inertia and momentum to it.

12. Djuric, J. "Spinning Magnetic Fields," Journal of Applied Physics Vol. 46, No. 2 (Feb. 1975) p. 679-688
Comments: A model of charge based on spinning magnetic dipoles is presented. The electric field of charge itself is claimed to be due to relative motion of magnetic flux. It is claimed that magnetic flux moves with it's source, even in axially symmetric rotating sources. This is directly contrary to the conclusions of Cohn. A historical summary of the debate whether flux rotates with its source or not is given p. 685-688.

13. Graneau, P. Ampere-Neumann Electrodynamics of Metals. Nonantum, MA. Hadronic Press, Inc. 1985.
Comments: During the IEEE Tesla Symposium (Aug., 1986), Graneau mentioned in private conversation, that an experiment had been done where a charged object reacted to steady state current flow in a metallic conductor.

14. Peters, P. "In What Frame is a Current-carrying Conductor Neutral?" American Journal of Physics Vol. 53, No. 12 (Dec. 1985) p. 1165-1169.
Comments: Uses special relativity to transform a current-carrying conductor and determine the reference frame in which it is neutral and finds that it is not the rest frame.

15. Moon, P. and Spencer D. "On the Ampere Force," Journal of Franklin Institute. Vol. 260 (1955) p. 295-311
Comments: Derives a general expression that matches Ampere's experimental results. Determines that a force exists between a neutral current and a stationary charge that is at variance with classical electromagnetism, but can not be eliminated by any choice of expression that covers the Ampere force comprehensively.

16. Price, P.B. "Evidence for the Detection of a Moving Magnetic Monopole," Physical Review Letters. 35 (8), Aug. 25, 1975. p. 487-490.

17. Greenberger, D.M. "Reality and Significance of the Aharonov-Bohm Effect," Phys. Rev. D. Particles and Fields. 23(6), Mar. 15, 1981. p. 1460.
Comments: The Aharonov-Bohm effect is the existence of measurable effects due to the magnetic vector potential in areas where the magnetic field is zero.

18. Calkin, M.G. "Linear Momentum of Quasistatic Electromagnetic Fields," American Journal of Physics. 34(10), Oct. 1966. p. 921-925.

19. Begley, S. "A Fifth Force in Physics, A study Offers an Amendment to the Law of Gravity," Newsweek. Jan. 20, 1986. p. 64.

20. Thompson, D.E. "The Attraction and Repulsion of Gravity," Science News. 129(17), May 6, 1986. p. 262

21. Graneau, P. "Electromagnetic Jet Propulsion in the Direction of Current Flow," Nature. 295, Jan. 28, 1982. p. 311-312.

22. Skitek, G. Electromagnetic Concepts and Applications. Englewood Cliffs, NJ. Prentice-Hall, Inc. 1982 p. 312, 262.
Comments: Definition of motional electric field is given on page 312 and the Biot-Savart law on page 262.

23. Mosca, E. "Magnetic Forces Doing Work". American Journal of Physics. 24 April, 1974 p. 295-297.

24. Lamb, T. "Unique Properties of Motional Electric Fields," EE 470 Special Topics. Montana State University, 1985.
Comments: Work preformed by J. C. Stover and K. A. Klicker on experiments with shielding motional electric fields and induction in field free regions summarized in paper by T. Lamb.

25. Maxwell, J. C. A treatise on Electricity and Magnetism. Vol. 2, p. 181.

26. Hooper, W.J. "All-electric motional electric field generator," U.S. Patent no. 3610971. Oct. 5, 1971.

27. Hooper, W.J. "The Motional Electric Field Generator," paper presented at the Southeastern sectional meeting of The American Physical Society. Nov. 8, 1969.

28. Hooper's private notes Feb. 4, 1969.

29. Gibson, F.G. "The All-Electric Motional Electric Field Generator and its Potential," Electrodynamic Gravity, Inc. Akron, OH 1983.

30. Hooper, W.J. "Similarities of the Motional Electric and Gravitational Fields." Akron OH: Electrodynamic Gravity, Inc. 1974. p. 7.